Activity Name	The Four Square Problem
Activity Description	Students are provided with a building mat with a long rectangle divided into nine equal squares and 4 centimeter cubes in each of two colors and instructed to complete the problem by asking only yes or no questions. These directions are posted on the board. The object of the problem is to start with each of the four cubes of each color placed at each end of the building mat leaving one space in the middle. [R: Red cube G: Green cube] Using only legal moves, move all of the reds from the left side of the board to the right, and all of the greens from the right side of the board to the left. | R | R | R | R | | G | G | G | G | Legal moves (and rules) include: • move only one cube at a time. • cubes can not move backwards • all eight cubes must remain on the board – unless it is in the process of being moved. This means you can't take cubes off to make more empty space. • No more than one cube may occupy a space at the same time. • a cube may slide forward into an adjacent empty space • a cube may jump forward over one other cube into an empty space, as long at the cube being jumped is of a different color. This also means you can not jump two or more cubes, or jump over the same color. • Yes this is possible.
Standards Addressed	This activity is rich with the Standards of Mathematical Practice. **Practice 1: Make sense of problems and persevere in solving them.** **Practice 3: Construct viable arguments and critique the reasoning of others.** **Practice 6: Attend to precision.** **Practice 8: Look for and express regularity in repeated reasoning.** **Practice 7: Look for and make use of structure.** Algebra I Mathematics Content Standard Functions -> Interpreting Functions (F-IF) **Analyze functions using different representations.** F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. MA.8.c. Translate among different representations of functions and relations: graphs, equations, point sets, and tables.
Materials	Each student requires: • a building mat • four cubes each of two different colors. Generally everyone in the class uses the same two colors.

Set-up	before class create collections of the cubes rather than count them out during class. I use Petri dishes to distribute cubes. No particular class set up is required, but students sitting side-by-side works well.
Prerequisite activities or prior knowledge required	none
Process	1. Distribute building mats and cubes to each student 2. Post the directions 3. Begin to answer questions, try to stick to yes or no answers. If asked a question apply it to the current state of a students progress. The answer might not be "yes" or "no" all of the time, but it might be "yes" or "no" at that moment. 4. It will take a little time to have students ask enough of the right questions to have the problem initially set up. Once all students have all of the cubes in the right place to begin the problem tell them they have done a great job and now they are ready to solve the problem. Many students will think that once they have set up the problem that they are done; nope; its just the start. 5. Make sure students are listening to the questions of other students, and the answers. You will get the same questions over and over, its just the nature of it; 6. Once questions for all of the legal moves have been answers it is okay to summarize them all so students can begin to try to solve the problem. 7. I use a smartnotebook to post the directions that also have the cubes and the building mat so moves can be demonstrated as needed. 8. Monitor the frustration level, let them have some, maybe even a lot, but don't let them give up. If students are near their tolerance for frustration, go to step 9. 9. If students are truly stuck, set up a "1" square problem using one cube of each color on a building mat of three squares. This is easily done on the smarboard by drawing some quick vertical lines. It can be set up on a student's building mat with two pencils. Ask the students to solve the "1" square problem – which all students will be successful. Have students proceed to a "2" square problem. Here it may take a few tries but all students will be successful at the 2-square problem. Then have them move to a "3" square problem, and then try the four square problem again. This will clearly show them that to solve a complicated problem sometimes a good start will be to solve a simpler problem first. Students at first won't believe a solution to be possible, then some of them would be able to solve a 10-square problem, or 100 square problem with enough /time/space

Why this activity is selected for use	• The way I present it, it requires students to ask questions.
	• It is fun to work with manipulatives,
	• It does not look like math
	• It will seem hard to many students, and then they will solve it and feel that they have solved a really hard problem. Even if they don't solve all of it they will be successful at some of it.
	• It helps me get to know the students and how some of them think
	    ○ I get to use their names often.
	    ○ Some students will think very carefully before making each move;
	    ○ some students will not think things through clearly and make quick moves, succeed or fail and try again. Some of the students that fail will figure out a pattern to where they get stuck, others will make the same mistakes over and over until the slow down.
	    ○ Some students will keep trying the four square problem while I'm asking the class to try the 1-square and 2-square problem because they think they are on the right track and want to stay with it.
	• It helps me to make students comfortable asking questions
	• Often the students that are not great with calculations and don't feel successful in math are the first ones to get this right and it builds their confidence and gives them a feel for being the smart kid.
	• If uncovers and applies the standards of mathematical practice without even using numbers.
	
	After completing this activity, will a student go home and say they haven't done any math? Maybe. Maybe "math" to some students means calculations. I'm trying to get students to start thinking of math as more than that.
	Mom: How was math today?
	Student: We didn't do any
	Mom: But I thought you were having math today
	Student: I did have math today, but we didn't do any, we just played a game.
	Mom: I'd better call the school and get you out of that math class, or even better, get the teacher fired while it is still early in the year.

An in depth discussion on why this activity is <u>great,</u> and how it really uncovers the richness of the Standards of Mathematical Practice.

Practice 1: Make sense of problems and persevere in solving them.

Big idea: Identify entry points to a solution

Student behavior
Students start understanding a problem by explaining to themselves what a problem is asking, what relevant information is being provided, and identify entry points toward its solution.

Students must ask questions or listen to the questions of others in order to figure out what to do and how to proceed. Often students will need to dive deeper into the meaning of a problem than what is provided in the directions. This problem exaggerates this skill.

Big idea: Plan a solution pathway

Student behavior
Students generate examples, and non-examples, and look for what successful examples have in common. They make conjectures about the solution form and meaning.

Students will realize that at any given point in the problem there are at most two possible legal moves. Students will begin to think several moves ahead using what-if scenarios, and will reduce the number of mistakes they make until the learn how to solve the problem.

Big idea: Monitor Progress, change course if needed

Student behavior
maintain sight of the goal.

Students will realize that after they have made several moves, if they end up with two cubes of the same color next to each other that they will not solve the problem. Their current solution path will allow them to make a few more legal moves, but that soon they would a have made moves that would not lead to a solution. After a few tries of solving the problem students will realize when they have made a mistake, and they will start over before exhausting all of the remaining moves. Students that can recognize when they have made a mistake are on the pathway toward a solution.

Big idea: **persevere**

Student behavior
perseverance

Students will get frustrated, not all at the same pace. Teacher interventions can prevent this from hitting critical mass. As students in the class experience success, even if they can’t repeat it yet, the will provide encouragement to frustrated students that will stick with the problem with more concrete assurance that a solution is possible.

Practice 3: Construct viable arguments and critique the reasoning of others.

Big idea: Listen and Critique

Student behavior
Students listen to the statements of others and ask clarifying or probing questions. They explore beyond what is presented to uncover the truth of what is being presented or argued and provide examples to support arguments or counterexamples to contrast arguments.

Students will have to listen to the questions asked by other students, and listen for the answer. That has been discussed.

Students will watch how other students will try to solve the problem. I have observed a student (on more than one occasion) that has not yet figured out how to completely solve the problem identify a move made by another student as legal but one that will not lead to a solution. So as it applies to this Big Idea, the statement made by one student is the move that was just made, and the critiquing by the other student is being able to identify when a "mistake" was made, and why it is a mistake. This is one of the ways that students share information during this activity. The "spirit" in which this information is exchanged ranges from very helpful and cooperative to competitive (and occasionally u-un-sportsman like. I've done this activity in at least 17 classes, I've seen a lot of ways to share opinions)

Practice 6: Attend to precision.

Big idea: Use specific language/Communication

Student behavior
description Mathematical statements are clear and unambiguous. At any moment, it is clear what is known and what is not known. (see Wu) Students critique their use of words and the implied definitions of vocabulary used by others, seek counterexamples and contradictions to deepen their understanding precise use of mathematics vocabulary and symbols.

Often students will demonstrate a move to me and ask me if it is good, or a legal move and I will answer it. The move might demonstrate a really important rule or legal move. I'll explain that someone in the back of the room can not and did not see the move, and could the student rephrase their question in such a way that it very specific so that a yes or no answer will have meaning to everyone in the class. When we talk about this activity in the next class – just to recall what we have done, I point this exact exchange out, and I talk about mathematical practice 6: Attend to precision. The practice does include rounding a number to the specified number of decimal places, and including units when we find the length or area of something, but it also includes using precise language.

Often students ask questions for which a yes or no answer does not make sense, or provides no real help. For example, "does the green cube go in the first square or an orange cube?" I respond "yes", playfully. Or a student says "I don't get it," or "I'm stuck" or any one of a number of statements to which my reply of yes or no makes no sense; this is to get the students to ask precise questions so a precise answer makes sense.

Practice 8: Look for and express regularity in repeated reasoning.

Big idea: Look for repetition in calculations

Student behavior
• Noticing when the same calculation is being done over and over again • seeing a “rhythm” in the operations. • Looking for a calculation short cut

This practice is really cool for this activity. Often when we have students repeat things they doing it with numbers and the structure an patterns of numbers, which falls more in line with Practice 7: Look for and make use of structure. To solve this problem a rhythm is almost a requirement.

Students will realize that to solve this problem a pattern of slides and jumps is required. When students are faced with move a cube to the left or one to the right, they will probably have a bias as to which one they want to do. Mine is to move from the right to the left. To solve the problem students may have to make a move that isn’t their first natural move to make, but they will realize what the move is.

Generally it takes students a while to solve the four square problem one time, then it takes them a while to solve it the second time because they didn’t pay attention to all of their moves the first time. The third time usually takes a lot less than the second, and soon students that can solve it can do so better when the moves become more mechanical and in rhythm than mental.

Students may get half way through the problem, 100% on track, the board may look symmetrical, and they may pause before making the next move. I’ve seen it many times that a student that in on the solution path, has not made any mistakes, pause for a second too long – loose the rhythm, and start all over rather than try to figure out the next move.

I do have activities planned that do this same thing with numbers. We will repeat calculations with random numbers, not getting any closer to a solution and not trying to uncover the structure of a problem, but to uncover the rhythm. For example, solving a multi-step equation. “we always subtracted this number first, and then divided by this one.” Another groups of students will find a different rhythm that will lead to a solution, but it will be different than finding a general rule – it will uncover a pattern in the process. That is what this problem is about, finding the pattern in the process.

Since this problem may be the one that has got me in the most trouble I don’t mind spending a few hours writing about how great I think this problem is. When I get to the follow-up activity with the students in a few weeks I’ll probably think it is even better than I already do. But the time I spent documenting this one has taken away from the time I can spend now completing the documentation for Algebra, and the activities for Trigonometry and Statistics. But trust me, I know what I am doing there too and why I’m doing it.

The follow-up activity

Algebra I Mathematics Content Standard

Functions -> Interpreting Functions (F-IF)

Analyze functions using different representations.

F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MA.8.c. Translate among different representations of functions and relations: graphs, equations, point sets, and tables.

Have students work in groups (4 students). Make sure that there is at least one student in each group that can solve the Four Square Problem. This should not be too difficult to do.

Have students count how many legal moves it takes to solve a one-square problem, a two-square problem, a three-square problem, and the four-square problem. Provide them with a recording sheet or a poster-page to record their answer in a table, or graph. Have students try to write an expression for solving an n-square problem. For additional questions you could provide them with an “average” time that it takes to make a single move and ask them to calculate the amount of time to solve a 4-sqaure, 5-square, 100-square, 1000-square problem and generalize for an n-square problem.

Students are not asked to do this part during the initial four-square problem, so I have not documented the full lesson that goes with this. Because this uses numbers and number patterns it is deep into Mathematical **Practice 7: Look for and make use of structure.**