AP Calc AB

Differential Equations Lab

Name: ________________________

Introduction: In this activity, you will be considering 6 differential equations that describe some positive quantity y. You will be matching from four sets of cards: differential equations, descriptions of the differential equation, slope fields, and descriptions of the solutions. In addition, you will complete some analysis questions related to each differential equation.

Directions:

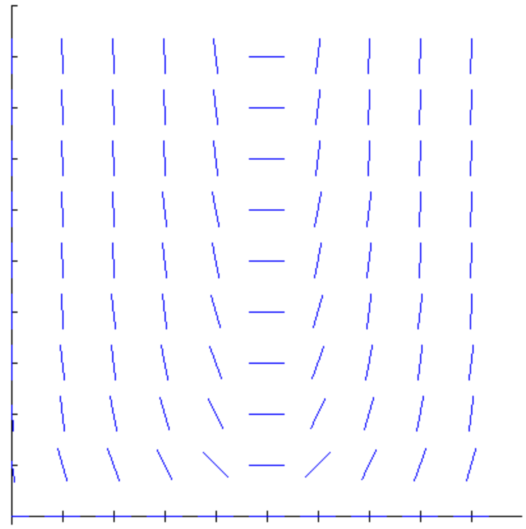
- Complete the matching and record your answers in the chart below.
- Check your results of the matching with the teacher.
- Once your matching has been verified, complete the additional analysis questions.

$\frac{dy}{dt}$	$\frac{dy}{dt}$ **Description**	**Slope Field**	**Solution Description**

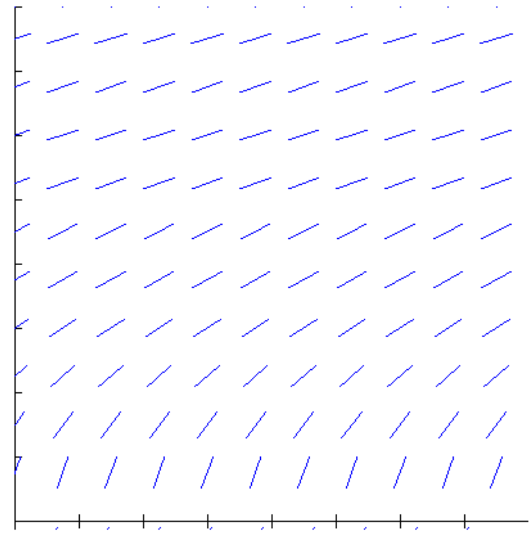
$\frac{dy}{dt}$ #1 $$\frac{dy}{dt} = 0.50y$$	$\frac{dy}{dt}$ #2 $$\frac{dy}{dt} = -0.25y$$
$\frac{dy}{dt}$ #3 $$\frac{dy}{dt} = -0.15(y-5)$$	$\frac{dy}{dt}$ #4 $$\frac{dy}{dt} = \frac{2}{y}$$
$\frac{dy}{dt}$ #5 $$\frac{dy}{dt} = y(t-5)$$	$\frac{dy}{dt}$ #6 $$\frac{dy}{dt} = 0.25y(4-y)$$

$\frac{dy}{dt}$ Description #1 y changes at a rate inversely proportional to the amount present at any time t	$\frac{dy}{dt}$ Description #2 At any given time, y changes at a rate proportional to the difference between the amount present and 5
$\frac{dy}{dt}$ Description #3 y increases at a rate proportional to the amount present at any time t	$\frac{dy}{dt}$ Description #4 At any given time, y changes at a rate proportional to the product of the amount present and 5 less than t.
$\frac{dy}{dt}$ Description #5 At any time t, y changes at a rate proportional to the product of the amount present and the difference between 4 and the amount present	$\frac{dy}{dt}$ Description #6 y decreases at a rate proportional to the amount present at any time t

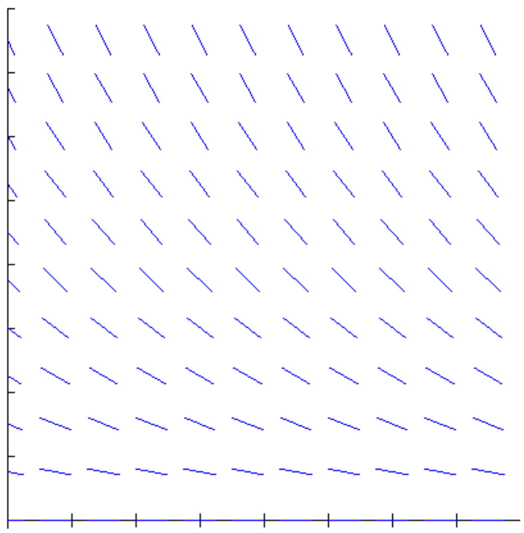
Slope Field #1



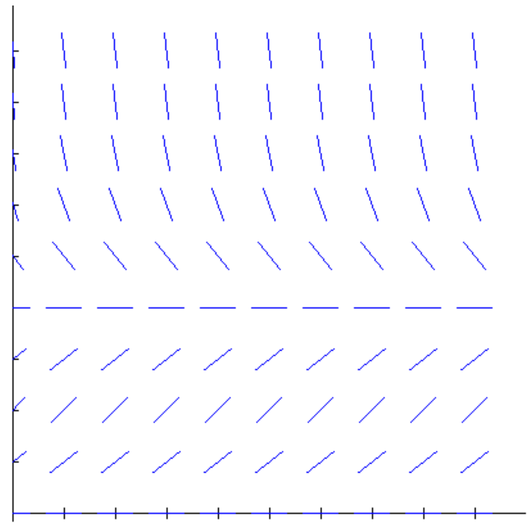
Slope Field #2



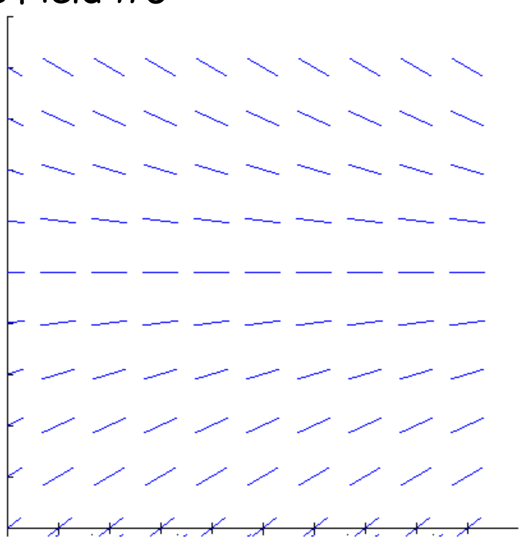
Slope Field #3



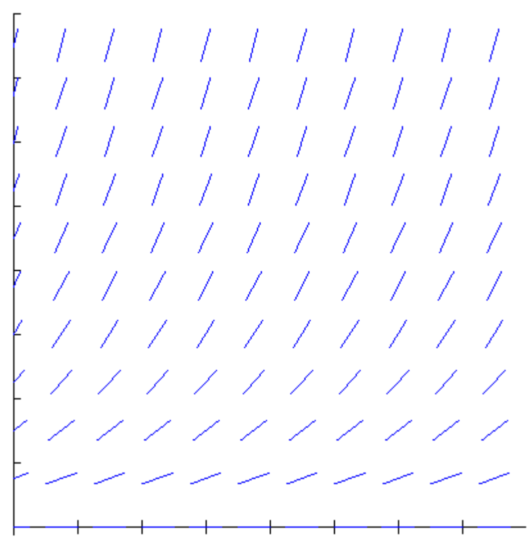
Slope Field #4



Slope Field #5



Slope Field #6



Solution Description #1 The solution curves are always concave down.	Solution Description #2 For any particular solution, $\lim_{t \to \infty} y(t) = 4$
Solution Description #3 The solution curves have horizontal tangents at $t = 5$.	Solution Description #4 For any particular solution, $\lim_{t \to \infty} y(t) = 0$
Solution Description #5 The solution curves represent exponential growth.	Solution Description #6 Each solution curve has a horizontal asymptote at $y = 5$.

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Differential Equations Lab Analysis Questions

1.) For the differential equation $\frac{dy}{dt}$ #1, suppose the initial condition is $y(0)=10$. How long until $y=100$?

2.) For the differential equation $\frac{dy}{dt}$ #2, find $\frac{d^2y}{dt^2}$ in terms of y. What does $\frac{d^2y}{dt^2}$ tell you about the solution curves for $y>0$?

3.) For the differential equation $\frac{dy}{dt}$ #3, use separation of variables to find the particular solution $y=y(t)$ with the initial condition $y(0)=2$.

4.) For the differential equation $\frac{dy}{dt}$ #4, compute $\frac{d^2y}{dt^2}$ in terms of y and use this justify why the solution curves are always concave down.

5.) For the differential equation $\frac{dy}{dt}$ #5, suppose that a particular solution $y = y(t)$ passes through the point (1, 2). Use the line tangent to the graph of $y = y(t)$ at $t = 1$ to estimate $y(1.1)$.

6.) For the differential equation $\frac{dy}{dt}$ #6, algebraically show that the function $y(t) = \frac{4}{1+e^{-t}}$ satisfies the differential equation.

Differential Equations Matching Teacher Answer Key

$\frac{dy}{dt}$	$\frac{dy}{dt}$ **Description**	**Slope Field**	**Solution Description**
1	3	6	5
2	6	3	4
3	2	5	6
4	1	2	1
5	4	1	3
6	5	4	2