CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Increasing and Decreasing Quantities by a Percent

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

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Increasing and Decreasing Quantities by a Percent

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to interpret percent increase and decrease, and in particular, to identify and help students who have the following difficulties:

- Translating between percents, decimals, and fractions.
- Representing percent increase and decrease as multiplication.
- Recognizing the relationship between increases and decreases.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.RP Use proportional relationships to solve multistep ratio and percent problems.
7.NS Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
7.NS Solve real-world and mathematical problems involving the four operations with rational numbers.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

2. Reason abstractly and quantitatively.
7. Look and make use of structure.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.
- Students work in small groups on collaborative discussion tasks, to organize percent, decimal and fraction cards. As they do this, they interpret the cards' meanings and begin to link them together. They also try to find relationships between percent changes. Throughout their work, students justify and explain their decisions to their peers.
- Students return to their original assessment tasks, and try to improve their own responses.

MATERIALS REQUIRED

- Each student will need, two copies of the assessment task *Percent Changes*, a calculator, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need copies of *Card Sets: A, B, C, D,* and *E*. All cards should be cut up before the lesson. Optional materials are a large sheet of card on which to make a poster, and some glue sticks and/or the poster template *Percents, Decimals, and Fractions (1)*.
- You will also need copies of the extension material: *Percents, Decimals, and Fractions (2)*.

There is also a projector resource to help with whole-class discussions.

TIME NEEDED

15 minutes before the lesson, one 90-minute lesson (or two 45-minute lessons), and 10 minutes in a follow-up lesson (or for homework). Timings are approximate and will depend on the needs of the class.

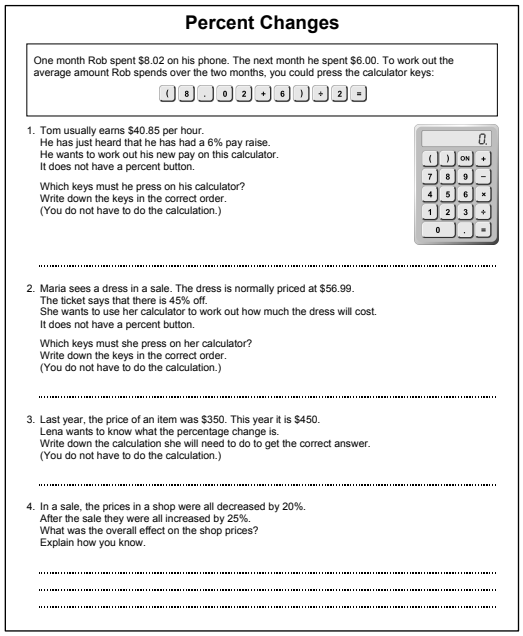
BEFORE THE LESSON

Assessment task: *Percent Changes* (15 minutes)

Have the students do this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Percent Changes.*

> *Read through the questions and try to answer them as carefully as you can. The example at the top of the page should help you understand how to write out your answers.*



Percent Changes

One month Rob spent $8.02 on his phone. The next month he spent $6.00. To work out the average amount Rob spends over the two months, you could press the calculator keys:

(8 . 0 2 + 6) ÷ 2 =

1. Tom usually earns $40.85 per hour.
   He has just heard that he has had a 6% pay raise.
   He wants to work out his new pay on this calculator.
   It does not have a percent button.

   Which keys must he press on his calculator?
   Write down the keys in the correct order.
   (You do not have to do the calculation.)

2. Maria sees a dress in a sale. The dress is normally priced at $56.99.
   The ticket says that there is 45% off.
   She wants to use her calculator to work out how much the dress will cost.
   It does not have a percent button.

   Which keys must she press on her calculator?
   Write down the keys in the correct order.
   (You do not have to do the calculation.)

3. Last year, the price of an item was $350. This year it is $450.
   Lena wants to know what the percentage change is.
   Write down the calculation she will need to do to get the correct answer.
   (You do not have to do the calculation.)

4. In a sale, the prices in a shop were all decreased by 20%.
   After the sale they were all increased by 25%.
   What was the overall effect on the shop prices?
   Explain how you know.

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help the majority of students. These can be written on the board at the end of the lesson.

The solution to all these difficulties is not to teach algorithms by rote, but rather to work meaningfully on the powerful idea that all percent changes are just multiplications by a scale factor.

Common issues:	Suggested questions and prompts:
Student makes the incorrect assumption that a percentage increase means the calculation must include an addition For example: 40.85 + 0.6 or 40.85 + 1.6. (Q1.) *A single multiplication by 1.06 is enough.*	• Does your answer make sense? Can you check that it is correct? • "Compared to last year 50% more people attended the festival." What does this mean? Describe in words how you can work out how many people attended the festival this year. Give me an example. • Can you express the increase as a single multiplication?
Student makes the incorrect assumption that a percentage decrease means the calculation must include a subtraction For example: 56.99 – 0.45 or 56.99 – 1.45. (Q2.) *A single multiplication by 0.55 is enough.*	• Does your answer make sense? Can you check that it is correct? • In a sale, an item is marked "50% off." What does this mean? Describe in words how you calculate the price of an item in the sale. Give me an example. • Can you express the decrease as a single multiplication?
Student converts the percentage to a decimal incorrectly For example: 40.85 × 0.6. (Q1.)	• How can you write 50% as a decimal? How can you write 5% as a decimal?
Student uses inefficient method For example: First the student calculates 1%, then multiplies by 6 to find 6%, and then adds this answer on: (40.85 ÷ 100) × 6 + 40.85. (Q1.) Or: 56.99 × 0.45 = ANS, then 56.99 – ANS (Q2.) *A single multiplication is enough.*	• Can you think of a method that reduces the number of calculator key presses? • How can you show your calculation with just one step?
Student is unable to calculate percentage change For example: 450 – 350 = 100% (Q3.) Or: The difference is calculated, then the student does not know how to proceed or he/she divides by 450. (Q3.) *The calculation (450 – 350) ÷ 350 × 100 is correct.*	• Are you calculating the percentage change to the amount $350 or to the amount $450? • If the price of a t-shirt increased by $6, describe in words how you could calculate the percentage change. Give me an example. Use the same method in Q3.
Student subtracts percentages For example: 25 – 20 = 5%. (Q4.) *Because we are combining multipliers: 0.8 × 1.25 = 1, there is no overall change in prices.*	• Make up the price of an item and check to see if your answer is correct.
Student fails to use brackets in the calculation For example: 450 – 350 ÷ 350 × 100. (Q4.)	• In your problem, what operation will the calculator carry out first?
Student misinterprets what needs to be included in the answer For example: The answer is just operator symbols.	• If you just entered these symbols into your calculator would you get the correct answer?

SUGGESTED LESSON OUTLINE

If you have a short lesson, or you find the lesson is progressing at a slower pace than anticipated, then we suggest you end the lesson after the first collaborative activity and continue in a second lesson.

Collaborative activity 1: matching *Card Sets A, B,* and *C* (30 minutes)

Organize the class into groups of two or three students. With larger groups some students may not fully engage in the task.

Give each group *Card Sets A* and *B*.

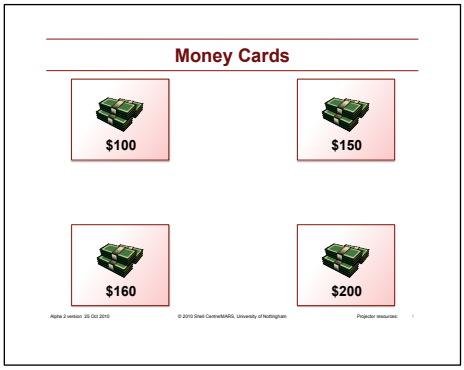
Use the projector resource to show students how to place *Card Set A*.

Introduce the lesson carefully:

I want you to work as a team. Take it in turns to place a percentage card between each pair of money cards.

Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.

There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.



Pairs of money cards may be considered horizontally or vertically.

Your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving

Make a note of student approaches to the task

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may make the mistake of pairing an increase of 50% with a decrease of 50%.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

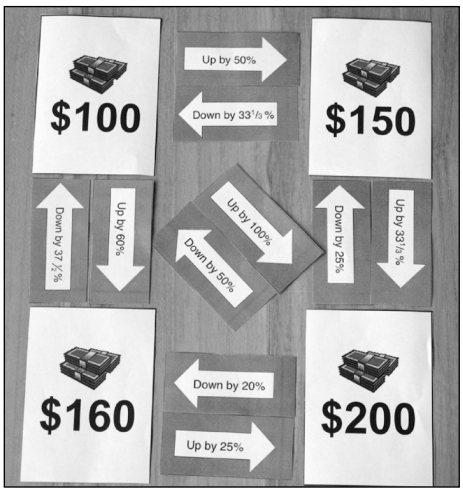
Students will correct their own errors once the decimal cards are added.

For students struggling to get started:

There are two ways to tackle this task. Can you think what they are? [Working out the percentage difference between the two money cards or taking a percentage card and using guess and check to work out where to place it.]

How can you figure out the percentage difference between these two cards?

This percentage card states the money goes up by 25%. If this money card (say $160) increases by 25% what would be its new value? Does your answer match any of the money cards on the table?



When one student has placed a particular percentage card, challenge their partner to provide an explanation.

Maria placed this percentage card here. Martin, why does Maria place it here?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common issues* table to support your questioning.

Students often assume that if an amount is increased and then decreased by the same percent, the amount remains unchanged.

The price of a blouse is $20. It increases by ½. What is the new price? [$30]

The price of the blouse now decreases by ½. What is the final price? [$15]

Now let's apply this to percentages. What happens if the $20 blouse increases by 50%?

What happens now when this new price decreases by 50%?

What percentage does the price need to decrease by to get it back to $20? [33⅓%]

What does this show?

If the whole class is struggling on the same issue, you may want to write a couple of questions on the board and organize a whole-class discussion. The projector resource may be useful when doing this.

It may help some students to imagine that the money cards represent the cost of an item, for example, the price of an MP3 player at four different stores.

Placing Card Set C: *Decimal Multipliers*

As students finish placing the percentage cards hand out Card Set C: *Decimal Multipliers*. These provide students with a different way of interpreting the situation.

Do not collect *Card Set B*. An important part of this task is for students to make connections between different representations of an increase or decrease.

Encourage students to use their calculators to check the arithmetic. Students may need help with interpreting the notation used for recurring decimals, and in entering $1.\overline{3}$ as 1.33333333 on the calculator.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. The following patterns may be noticed:

An increase of, say, 33% is equivalent to multiplying by $1.\overline{3}$.
(An increase of 5% is not equivalent to multiplying by 1.5!)
A decrease of, say, 33% is equivalent to multiplying by $(1-1.\overline{3}) = 0.\overline{6}$.
The inverse of an increase by a percent is not a decrease by the same percent.

When the decimal multipliers are considered in pairs, the calculator will show that each pair multiplies to give 1, subject to rounding by the calculator.

× 2	× 0.5	and 2 × 0.5 = 1
× 1.5	× $0.\overline{6}$	and 1.5 × $0.\overline{6}$ = 1
× $1.\overline{3}$	× 0.75	and $1.\overline{3}$ × 0.75 = 1
× 1.25	× 0.8	and 1.25 × 0.8 = 1
× 1.6	× 0.625	and 1.6 × 0.625 = 1

Extension activity

Ask students who finish quickly to try to find the percent changes and decimal multipliers that lie between the diagonals $150/$160 and $100/$200. Students will need to use the blank cards for the diagonals $150/$160.

Taking two lessons to complete all activities

You may decide to extend the lesson over two periods. Ten minutes before the end of the first lesson ask one student from each group to visit another group's work. Students remaining at their seats should explain their reasoning for the position of the cards on their own desk (see the section on Sharing Work for further details.)

When students are completely satisfied with their own work, hand out the poster template *Percents, Decimals, and Fractions (1)*. Students should use it to record the position of their cards. At this stage, one pair of arrows between each money card will be left blank.

At the start of the second lesson spend a few minutes reminding the class about the activity.

> *Try to remember what we were working on in the last lesson.*
>
> *A mobile phone is reduced by 60% in the sale. Give me an example of what the phone could have originally cost and what it costs now. And another, and another...*
>
> *[Take one of the examples given above.]*
> *The mobile phone is not sold. It returns to its original price. What is the percent increase?*

Return to each group their *Percents, Decimals, and Fractions (1)* sheet and the *Card Sets A, B* and *C*. Ask students to use their sheet to position their cards on the desk. Working with the cards instead of the sheet means students can easily make changes to their work and encourages collaboration between students.

Then move the class on to the second collaborative activity.

Sharing work (10 minutes)

When students get as far as they can with placing *Card Set C*, ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

> *If you are staying at your desk, be ready to explain the reasons for your group's matches.*
>
> *If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.*
>
> *If there are differences, ask for an explanation. If you still don't agree, explain your own thinking. When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.*

Students may now want to make changes.

Collaborative activity 2: matching *Card Set D* (30 minutes)

Give out *Card Set D: Fraction Multipliers*. These may help students to understand **why** the pattern of decimal multipliers works as it does.

Support the students as you did in the first collaborative activity.

The following pairings appear:

	and	$\times\frac{1}{2}$	$\times\frac{3}{2}$	and	$\times\frac{2}{3}$
$\times\frac{4}{3}$	and	$\times\frac{3}{4}$	$\times\frac{5}{4}$	and	$\times\frac{4}{5}$
$\times\frac{8}{5}$	and	$\times\frac{5}{8}$			

Sharing work (10 minutes)

When students get as far as they can placing *Card Set D*, ask the student who has not already visited another group to go check their answers against that of another group's work. As in the previous sharing activity, students remaining at their desk are to explain their reasoning for the matched cards on their own desk.

Students may now want to make some final changes to their own work. After they have done this, they can make a poster.

Either:

- Give each group a large sheet of paper and a glue stick, and ask students to stick their final arrangement onto a large sheet of paper

and/or:

- Give each group the poster template *Percents, Decimals, and Fractions (1)* and ask students to record the position of their cards.

The poster template allows students to record their finished work. It should not replace the cards during the main activities of this lesson as students can more easily make changes when working with the cards, and they encourage collaboration.

Extension activities

Ask students who finish quickly to try to find the fraction multipliers that lie between the diagonals \$150/\$160 and \$100/\$200.

Card Set E: Money Cards (2) may be given to students who need an additional challenge. *Card Sets B–D* can again be used with these *Money Cards*. Students can record their results on the poster template *Percents, Decimals, and Fractions (2)*.

In addition, you could ask some students to devise their own sets of cards.

Whole-class discussion (10 minutes)

Give each student a mini-whiteboard, pen, and eraser.

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions listed above. As you ask students questions like the following, they should respond using mini-whiteboards.

Suppose prices increase by 10%. How can I say that as a decimal multiplication?

How can I write that as a fraction multiplication?

What is the fraction multiplication to get back to the original price?

How can you write that as a decimal multiplication?

How can you write that as a percentage?

Improving individual solutions to the assessment task (10 minutes)

Return the original assessment, *Percentage Change*, to the students together with a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

SOLUTIONS

Assessment Task: *Percent Changes*

Students may answer Questions 1 - 3 in several ways. Here are some possible answers:

1. $40.85 \times 1.06 =$

   or $(40.85 \times 0.06) + 40.85 =$

   or $40.85 \times 0.06 =$ ANS, ANS $+ 40.85 =$

2. $56.99 \times 0.55 =$

   or $56.99 - (56.99 \times 0.45) =$

   or $56.99 \times 0.45 =$ ANS, $56.99 -$ ANS $=$

3. $(450 - 350) \div 350 \times 100 =$

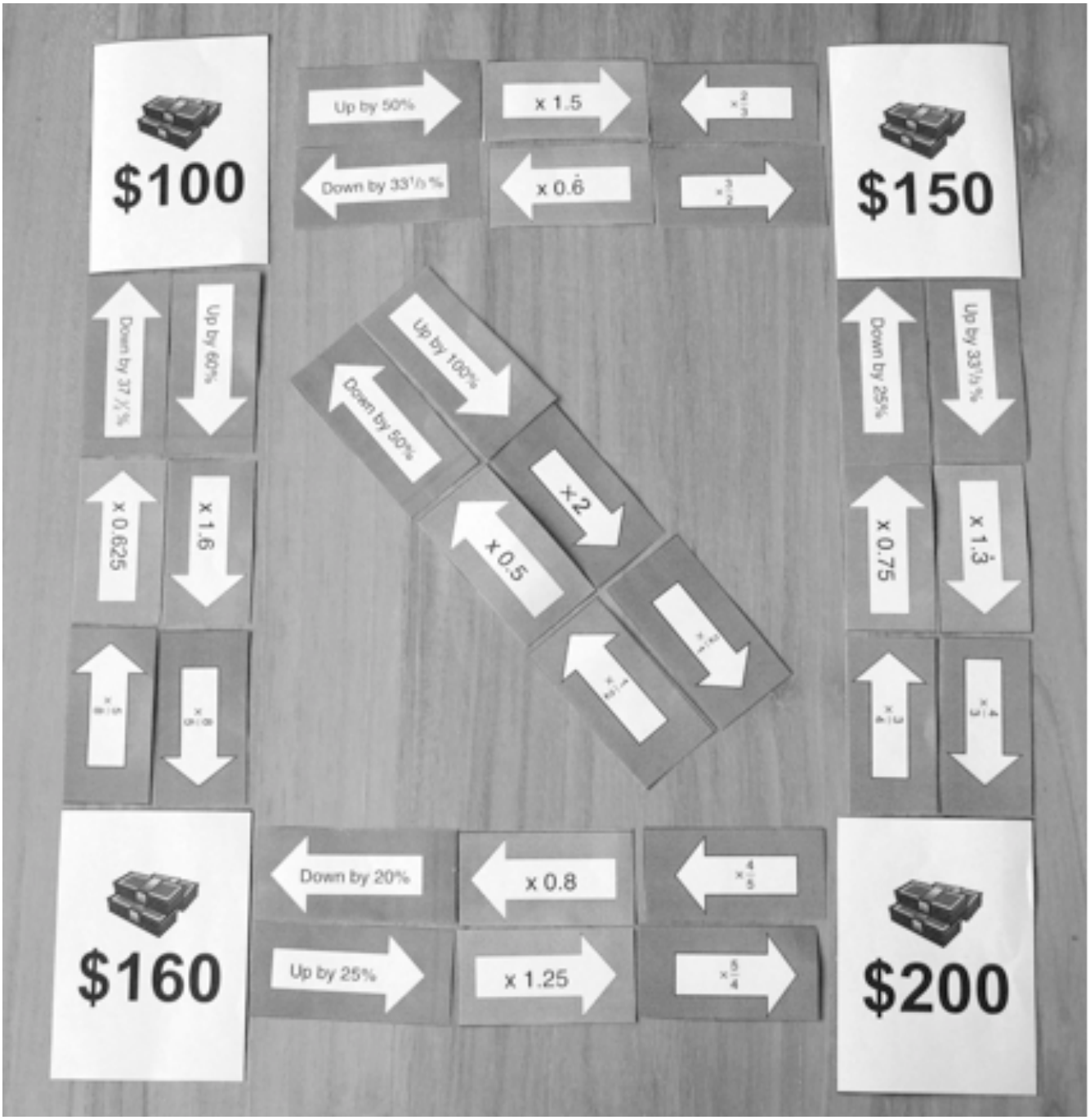
   or $450 - 350 =$ ANS, ANS $\div 350 \times 100 =$

4. There is no overall change in the price:

   cost of product $\times 0.8 \times 1.25 =$ cost of product **or**

   cost of product $\times \frac{4}{5} \times \frac{5}{4} =$ cost of product

Collaborative activity



Percent Changes

One month Rob spent $8.02 on his phone. The next month he spent $6.00. To work out the average amount Rob spends over the two months, you could press the calculator keys:

(8 . 0 2 + 6) ÷ 2 =

1. Tom usually earns $40.85 per hour.
   He has just heard that he has had a 6% pay raise.
   He wants to work out his new pay on this calculator.
   It does not have a percent button.

   Which keys must he press on his calculator?
   Write down the keys in the correct order.
   (You do not have to do the calculation.)

2. Maria sees a dress in a sale. The dress is normally priced at $56.99.
   The ticket says that there is 45% off.
   She wants to use her calculator to work out how much the dress will cost.
   It does not have a percent button.

   Which keys must she press on her calculator?
   Write down the keys in the correct order.
   (You do not have to do the calculation.)

3. Last year, the price of an item was $350. This year it is $450.
   Lena wants to know what the percentage change is.
   Write down the calculation she will need to do to get the correct answer.
   (You do not have to do the calculation.)

4. In a sale, the prices in a shop were all decreased by 20%.
   After the sale they were all increased by 25%.
   What was the overall effect on the shop prices?
   Explain how you know.

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Card Set A: Money Cards (1)

$100

$150

$200

$160

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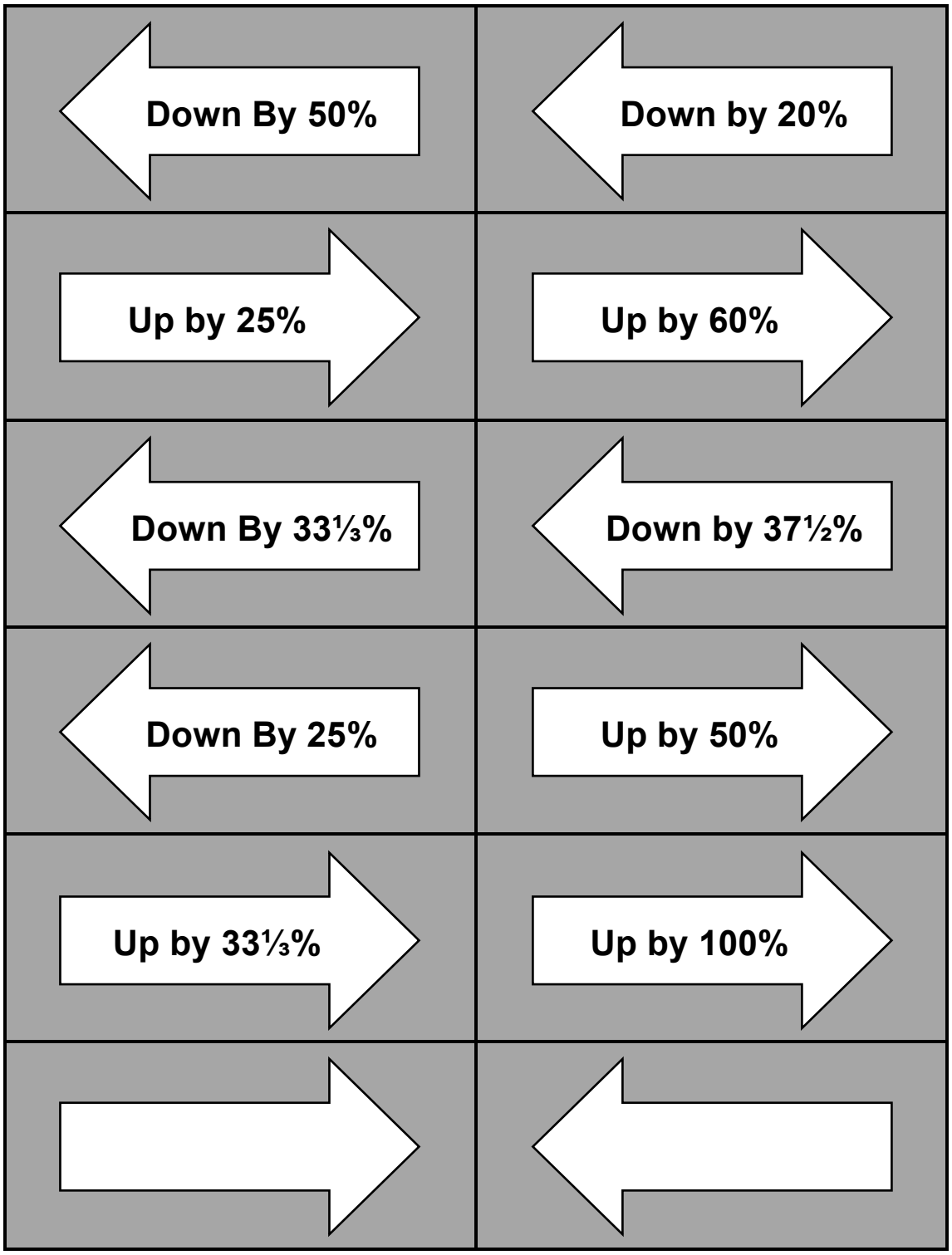
Card Set B: Percent Increases and Decreases



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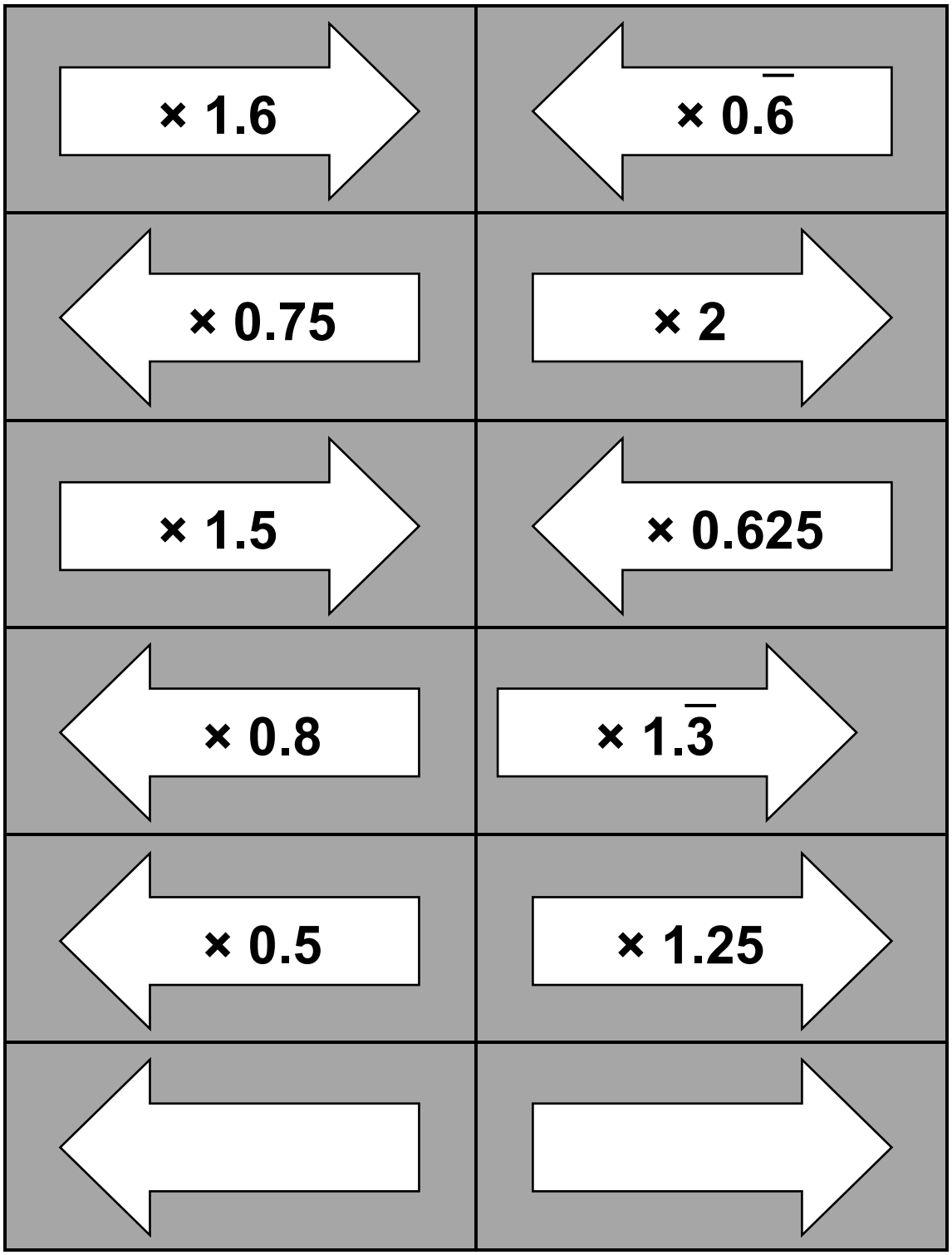
Card Set C: Decimal Multipliers



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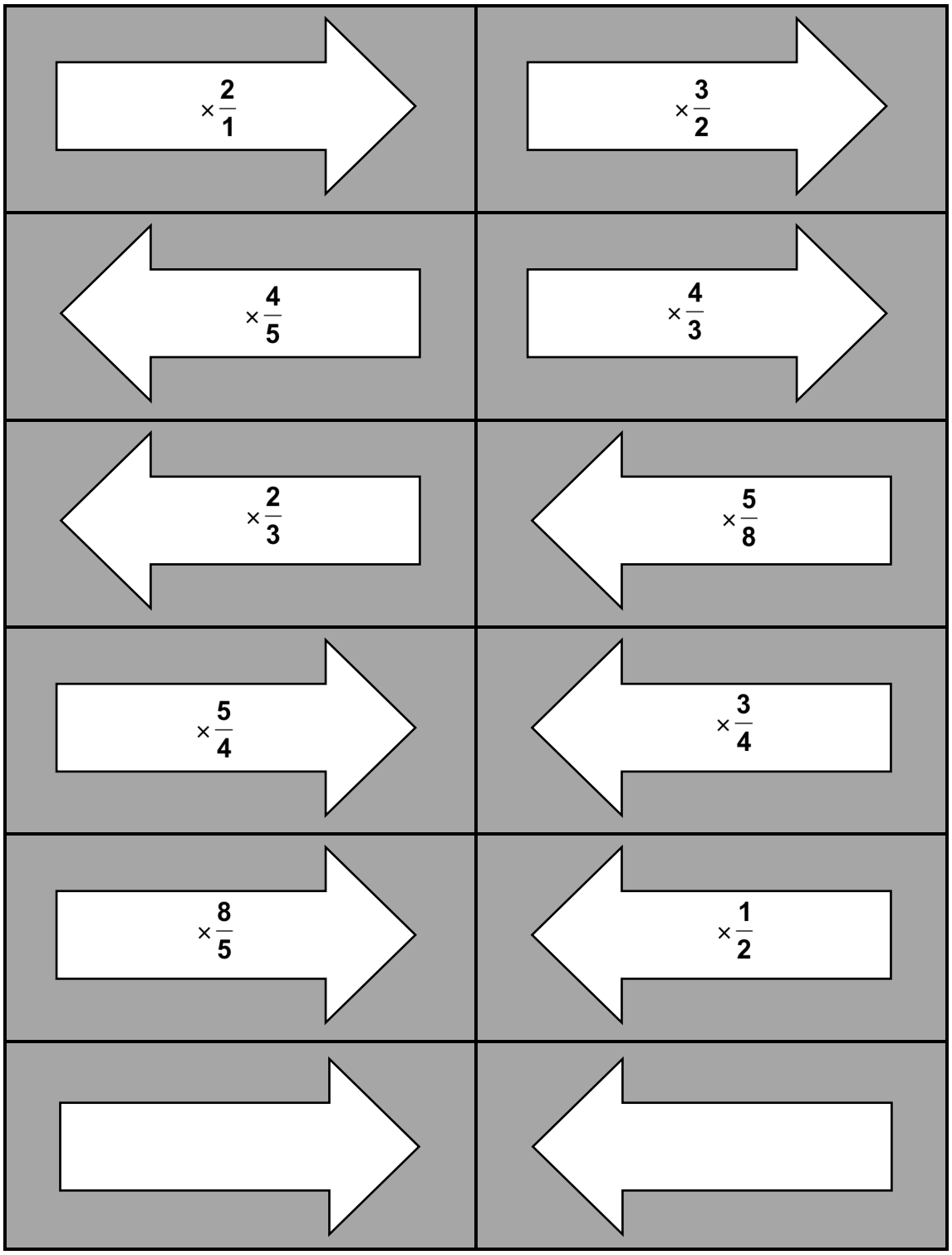
Card Set D: Fraction Multipliers



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Card Set E: Money Cards (2)

80¢

$1.20

$1.60

$1.28

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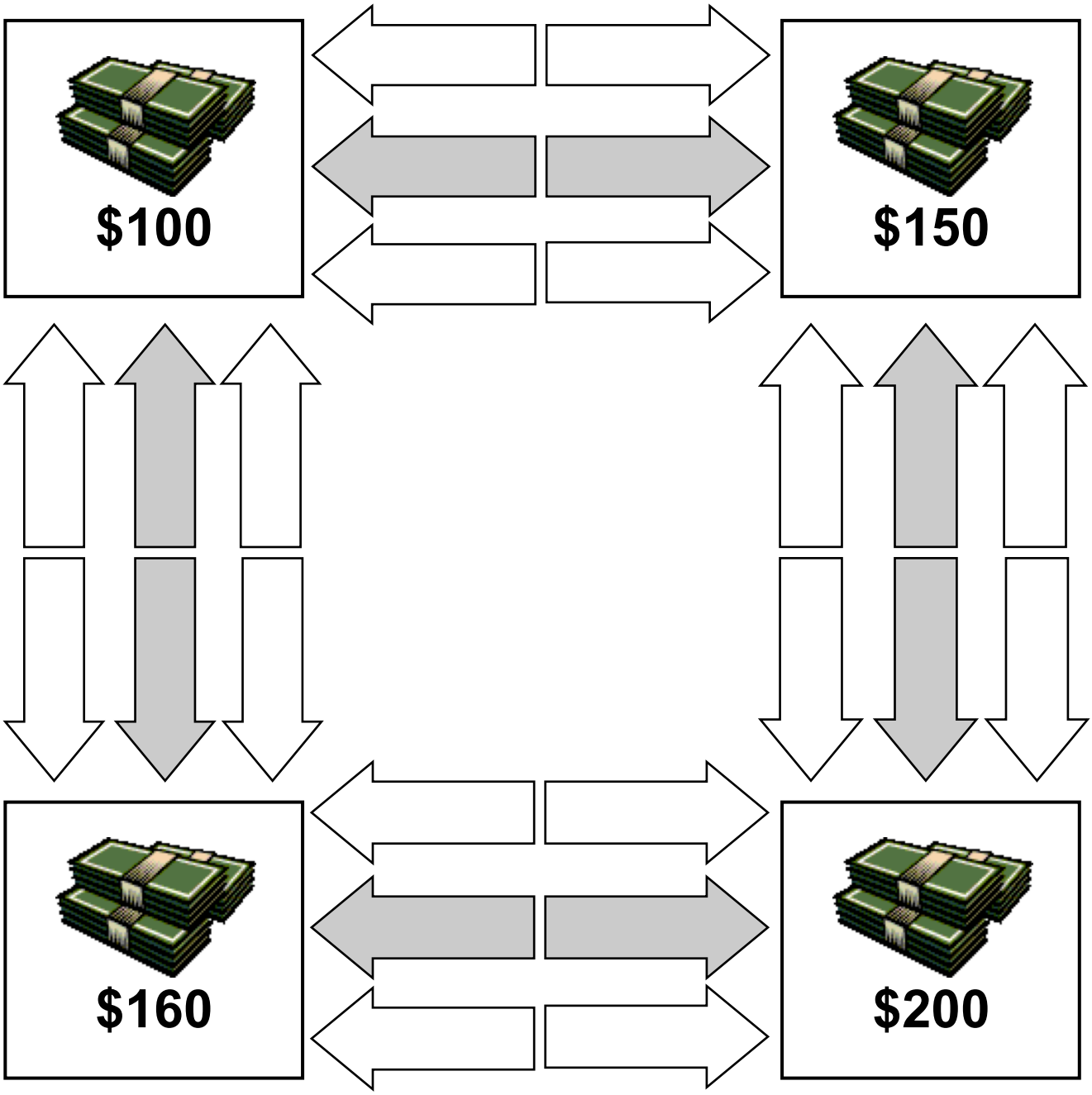
Percents, Decimals, and Fractions (1)



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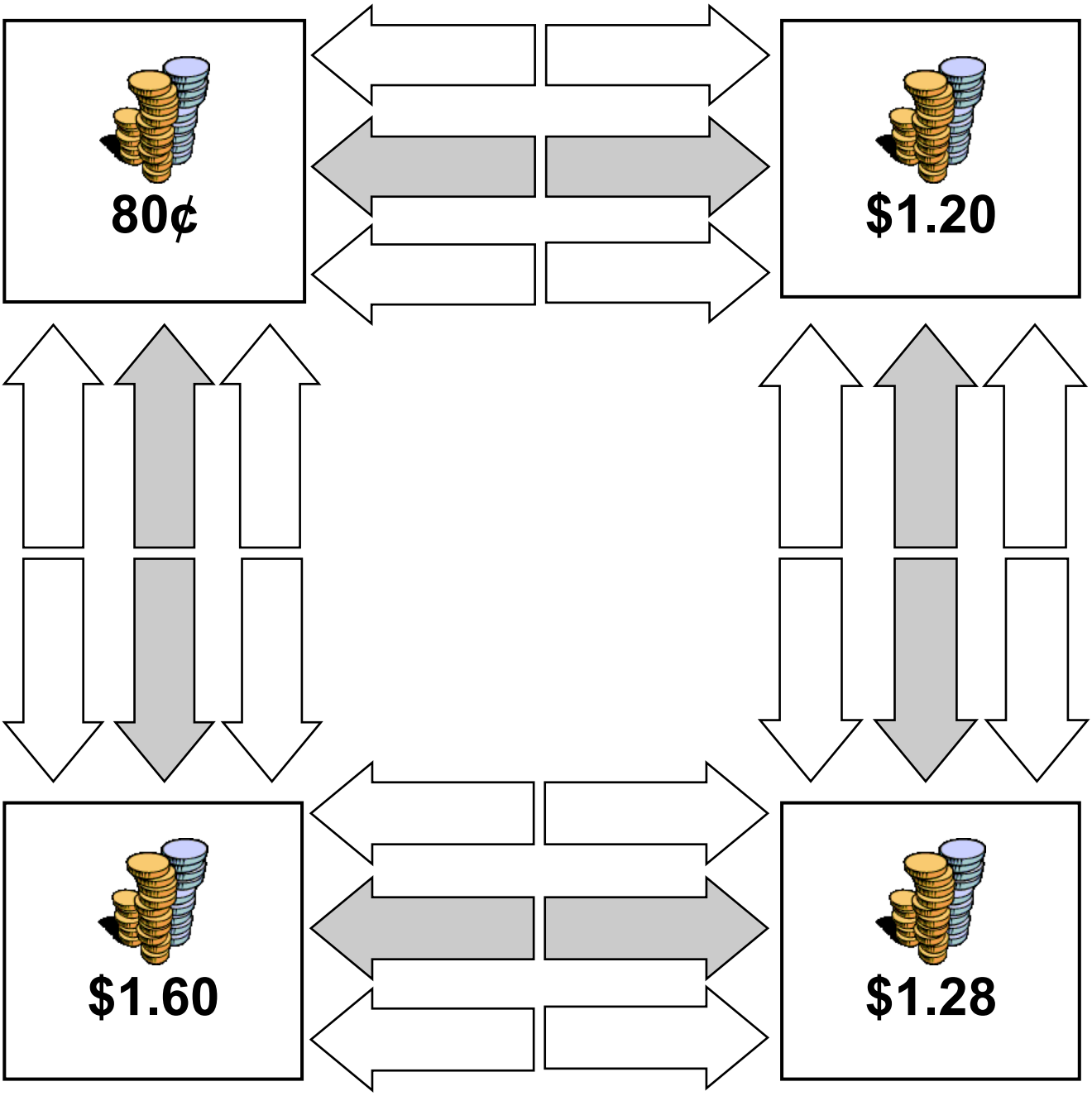
Percents, Decimals, and Fractions (2)



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Money Cards

$100

$150

$160

$200

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
at the
University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by
Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

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